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$\mathcal{N}=8$ SCFT and M theory on $AdS_4 \times RP^7$

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We study M theory on $AdS_4 \times RP^7$ corresponding to 3 dimensional $\mathcal{N}=8$ superconformal field theory which is the strong coupling limit of 3 dimensional super Yang-Mills theory. For $SU(N)$ theory, a wrapped M5 brane on RP^5 can be interpreted as the baryon vertex. For $SO(N)/Sp(2N)$ theory, by using the property of (co)homology of RP^7, we classify various wrapping branes and consider domain walls and the baryon vertex.

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I. INTRODUCTION

In [1] the large N limit of superconformal field theories (SCFT) was described by taking the supergravity limit on anti–de Sitter (AdS) space. The scaling dimensions of operators of SCFT can be obtained from the masses of particles in string or M theory [2]. In particular, $\mathcal{N}=4$ $SU(N)$ super Yang-Mills (SYM) theory in 4 dimensions is described by type IIB string theory on $AdS_5 \times S^5$. This AdS-CFT correspondence was tested by studying the Kaluza-Klein (KK) states of supergravity theory and by comparing them with the chiral primary operators of SCFT on the boundary [3]. There exist also $\mathcal{N}=2$, 1, 0 superconformal theories in 4 dimensions which have a corresponding supergravity description on orbifolds of $AdS_5 \times S^5$ [4] and the KK spectrum description on the twisted states of AdS_5 orbifolds was discussed in [5]. The energy of a quark-antiquark pair [6], glueball mass spectrum [7] and the energy of a baryon as a function of its size [8,9] were analytically calculated based on this correspondence. The field-theory–M theory duality also provides a supergravity description on AdS_4 or AdS_7 for some superconformal theories in 3 or 6 dimensions, respectively [1]. The maximally supersymmetric theories have been studied in [10,11] and the lower supersymmetric case was also realized on the world volume of M theory at orbifold singularities [12].

The gauge group of the boundary theory becomes $SO(N)/Sp(2N)$ [13] by taking appropriate orientifold operations for the string theory on $AdS_5 \times S^5$ (see also [10,14]). By analyzing the discrete torsion for B fields, the possible models of gauge theory are topologically classified and many features of gauge theory are described by various wrapping branes. By generalizing the work of [13] to the case of $AdS_7 \times RP^4$ where the eleventh dimensional circle is one of AdS_7 coordinates, (0, 2) six dimensional SCFT on a circle rather than uncompactified full M theory was described in [15]. For $SU(N)$ (0, 2) theory, a wrapped D4 brane on S^4 together with fundamental strings connecting a D4 brane on the boundary of AdS_7 with the D4 brane on S^4 was interpreted as baryon vertex. By putting N M5 branes in the R^5/Z_2 orbifold singularity [16], where the Z_2 acts by a reflection of the 5 directions transverse to the M5 branes and also by changing the sign of the 3-form field C_3, the large N limit of the $SO(2N)$ (0,2) SCFT and RP^4 orientifold after removing the R^5/Z_2 orbifold singularity were obtained. Then, using the property of (co-)homology of $RP^i \subset RP^4$, various wrapping branes and their topological restrictions were discussed.

Recently, Sethi [17] found that $\mathcal{N}=8$ $Sp(2N)$ and $SO(2N+1)$ gauge theories in 3 dimensions flow to the same strong coupling fixed point. This was confirmed by turning on discrete torsion of M2 branes on R^8/Z_2. By evaluating $C_3 \wedge G_4$ over RP^7 where $G_4 = dC_3$ is four-form field in M theory, it was shown that there exists M2 brane charge shift for the two types of orientifold two-plane. This result implies that in IR limit three dimensional $\mathcal{N}=8$ SYM theories can flow to two distinct strong coupling conformal field theories.

In this paper, we generalize the work of [13,15] to the case of $AdS_4 \times RP^7$ where the eleventh dimensional circle is one of RP^7 coordinates,[1] as we briefly mentioned this possibility in the previous paper [15]. In Sec. II, it will be shown that for $SU(N)$ theory, a wrapped M5 brane on RP^5 can be interpreted as baryon vertex [18]. By putting N M2 branes in the $R^8/\mathbf{Z}_2$ orbifold singularity, where the Z_2 acts by a reflection of the 8 directions transverse to the M2 branes, we will

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[1]Although the eleven dimensional solution by uplifting the D2 brane solution is not exactly M2 brane solution in general, when the eleventh dimension is compact, by taking M2 branes to be localized in the eight transverse dimensions, they will resemble each other more and more in M theory limit [19,20].

obtain the large N limit of $\mathcal{N}=8$ SCFT on RP^7 orbifold [10]. Then, using the property of (co-)homology of $RP^i \subset RP^7$, we classify various wrapping branes and discuss their topological restrictions. In Sec. III, we consider domain walls and the baryon vertex. Finally, in Sec. IV, we will discuss important open problems and comment on the future directions.

II. SO/Sp SYM AND BRANES ON RP^7

A. The baryon vertex in $SU(N)$

Let us consider M theory on $R^3 \times R^7 \times S^1$. For R^3, we can take (x^0,x^1,x^2) directions and for $\mathbf{R}^7$, we take $(x^3,x^4,x^5,x^6,x^7,x^8,x^9)$ transverse to M2 branes. The eleventh coordinate x^{10} is compactified on a circle S^1 and is a periodic coordinate of period 2π. For small radius of S^1, we can regard M theory as type IIA string theory which can be described in the context of $AdS_4 \times S^7$ where D2 branes are realized by transverse M2 branes. The radial function $\rho = \sqrt{\Sigma_{i=3}^{10}(x^i)^2}$ of $R^7 \times S^1$ will be one of the AdS_4 coordinates, the other three being the ones in R^3.

The $AdS_4 \times S^7$ compactification has N units of seven-form flux on S^7 as follows [8]:

$$\int_{S^7} \frac{G_7}{2\pi} = N, \qquad (2.1)$$

where G_7 is seven-form field which is a hodge dual of $G_4 = dC_3$ four-form field. By putting a large number of N of coincident M2 branes and taking the near horizon limit, the metric becomes that [18] of $AdS_4 \times S^7$

$$ds_{11}^2 = \frac{r^4}{L^{2/3}} \eta_{\mu\nu} dy^\mu dy^\nu + L^{1/3}\left(\frac{dr^2}{r^2} + g_{ij} dx^i dx^j\right). \qquad (2.2)$$

The scale L is related to N by [18]

$$L = \left(\frac{\Lambda}{6}\right)^{-3} = l_p^6 2^5 \pi^2 N, \qquad (2.3)$$

where l_p is a Planck scale which is the only universal parameter in M theory and $\mathrm{Vol}(S^7) = (\pi^4/3)(6/\Lambda)^{7/2}$.[2] The first equation arises when we write AdS_4 radius in terms of both cosmological constant Λ and scale factor L. Since M2 branes have the operators with dimension $\sqrt{N}$ by M2 tension formula and M5 branes have the operators with dimension N through the relation between mass, tension [21] and volume of branes, we consider wrapping a M5 brane over 5-cycle of S^7.

A 5-cycle of minimum volume is to take the subspace at a constant value of two coordinates. For the 5 volume, Vol(5-cycle), one can find $\mathrm{Vol(5\text{-}cycle)} = \mathrm{Vol}(S^5) = \pi^3(6/\Lambda)^{5/2}$. The mass of the M5 brane wrapped over 5-cycle, given by M5 brane tension times Vol(5-cycle), is

$$m = \frac{1}{(2\pi)^5 l_p^6} \mathrm{Vol}(5-\mathrm{cycle}). \qquad (2.4)$$

By the relation $m^2 = (2\Lambda/3)(\Delta-1)(\Delta-2) \approx (2\Lambda/3)\Delta^2$ for large Δ and the relations (2.4) and (2.3), one gets for the mass formula [18] for the dimension of a baryon corresponding to the M5 brane wrapped 5-cycle

$$\Delta = \frac{\pi N}{\Lambda} \frac{\mathrm{Vol}(S^5)}{\mathrm{Vol}(S^7)} = \frac{N}{2}. \qquad (2.5)$$

B. The RP^7 orientifold

It was observed [10] that the large N limit of $\mathcal{N}=8$ SCFT in 3 dimensions corresponds to N M2 branes coinciding at R^8/Z_2 orbifold singularity. Let us consider M theory on[3] $R^3 \times (R^7 \times S^1)/Z_2$. The Z_2 acts by sign change on all eight coordinates in R^7 and S^1 as follows: $(x^3,\cdots,x^{10}) \to (-x^3,\cdots,-x^{10})$. This gives two orbifold singularities at $x^{10}=0$ and $x^{10}=\pi$, each of which locally looks like R^8/Z_2. The angular directions in R^8/Z_2 are identified with RP^7. Consider N parallel M2 branes which are sitting at an orbifold two-plane (O2-plane) which is located at $x^3 = \cdots = x^{10} = 0$. Note that there is another singularity at $x^{10}=\pi$ but we will focus on the theory at the origin which has corresponding interacting superconformal field theory [22]. We will describe how $\mathcal{N}=8$ SCFT in 3 dimensions can be interpreted as M theory on $AdS_4 \times RP^7$ where the eleventh dimensional circle is in RP^7 space. This is our main goal in this paper.

Let us study the property of $AdS_4 \times RP^7$ orientifold. Let x be the generator of $H^1(RP^7, Z_2)$ which is isomorphic to Z_2, Σ be a string worldsheet and $w_1(\Sigma) \in H^1(\Sigma, Z_2)$ be the obstruction to its orientability. Then we only consider the map $\Phi: \Sigma \to AdS_4 \times RP^7$ such that $\Phi^*(x) = w_1(\Sigma)$. Since Z_2 action on S^7 is free (no orientifold fixed points), there is no open string sector. In the orientifold the string world sheet need not be orientable and a basic case of an unorientable closed string worldsheet is $\Sigma = RP^2$, which can be identified with the quotient of the two sphere S^2 by the overall sign change. The map $\Phi: RP^2 \to RP^7$ satisfying the constraints $\Phi^*(x) = w_1(RP^2)$ is the embedding $(x_1,x_2,x_3) \to (x_1,x_2,x_3,0,0,0,0,0)$.

In M theory, there is the Chern-Simons interaction in eleven dimensional supergravity:

[2] The normalization [18] for four-form field strength is $G_{ijkl} = e\epsilon_{ijkl}$ where the parameter e is a real constant. By plugging this into the 11 dimensional field equations, it leads to the product of 4 dimensional Einstein space, $R_{\mu\nu} = -2\Lambda\eta_{\mu\nu}$ with Minkowski signature$(-,+,+,+)$ and 7 dimensional Einstein space $R_{ij} = \Lambda g_{ij}$ where Λ is defined by $\Lambda = 24e^2/\kappa^{4/9}$ through grvitational constant κ. Moreover $\kappa^2 = 8\pi G_{11} = (2\pi)^8 l_p^9/2$.

[3] The eleven dimensional spacetime is not a principal $U(1)$-bundle over 10 dimensional spacetime although it can be defined as a $U(1)$-bundle over 10 dimensional spacetime. However, it will still make sense to consider Ramond-Ramond (RR) $U(1)$ gauge field as one-form with values in the twisted bundle. We thank K. Hori for pointing out this.

$$-\frac{1}{24\pi^2}\int C_3\wedge G_4\wedge G_4, \tag{2.6}$$

where $G_4=dC_3$. A compactification of M theory on an eightfold X_8 receives tadpole contribution for the C_3 three-form field in one loop [23,24]

$$-\int_{X_8} C_3\wedge I_8(R), \tag{2.7}$$

where $J=-\int_{X_8}I_8(R)=\chi/24$, with χ the Euler characteristic of the eightfold X_8 and $I_8(R)$ is an eight-form constructed as a quartic polynomial in the curvature. The condition for a consistent M theory compactification on an eightfold is thus

$$\frac{\chi}{24}-\frac{1}{8\pi^2}\int_{X_8} G_4\wedge G_4-n=0, \tag{2.8}$$

where n is the number of M2 branes filling the vacuum. It can be deduced from the relation (2.8) the orbifold R^8/Z_2 carries $-1/8$ units of M2 brane charge [25].[4]

It was shown by Sethi [17] that there exist three O2 planes:[5]

$$\begin{aligned}&\text{(i)}\quad SO(2N)\quad\text{with}\quad O2^-:\quad \text{M theory on } R^3\times(R^7\times S^1)/Z_2,\\&\text{(ii)}\quad SO(2N+1)\quad\text{with}\quad \widetilde{O2}^+:\quad \text{M theory on } R^3\times(R^7\times S^1)/Z_2,\\&\text{(iii)}\quad Sp(2N)\quad\text{with}\quad O2^+:\quad \text{M theory on } R^3\times(R^7\times S^1)/Z_2.\end{aligned} \tag{2.9}$$

Here $O2^-$ characterized by R^7/Z_2 has $-1/4$ unit of D2 brane charge. By promoting to M theory, each fixed point (0 and π on the circle) realized by R^8/Z_2 carries $-1/8$ unit of M2 brane charge. We should have a single D2 brane stuck on the $O2^-$ plane to get $SO(2N+1)$ gauge group and the orientifold R^7/Z_2 carries 3/4 units of D2 brane charge. It is denoted by $\widetilde{O2}^+$. In strong coupling limit, $\widetilde{O2}^+$ plane splits into two orbifolds R^8/Z_2. Apparently the stuck M2 brane also splits into two fluxes, each carrying 1/2 unit of M2 brane charge. Each fixed point R^8/Z_2 has thus $3/8=-1/8+1/2$ units of M2 brane charge. As shown by Sethi [17], the M theory realization of the charge shift is given by the term

$$-\frac{1}{2}\int_{RP^7}\frac{C_3}{2\pi}\wedge\frac{G_4}{2\pi}=-\frac{1}{2}\int_{\mathcal{M}}\frac{G_4}{2\pi}\wedge\frac{G_4}{2\pi}=\frac{1}{4}, \tag{2.10}$$

where $G_4/2\pi$ is the torsion class and RP^7 is the boundary of the smooth eightfold $\mathcal{M}$. Finally we can obtain $O2^+$ plane by the basis choice of the Chan-Paton factors giving gauge group $Sp(2N)$, which has the 1/4 unit of D2 brane charge. In this case, the M theory interpretation on the orbifold singularity is a little different: the singularity at $x^{10}=0$ has the charge $3/8=-1/8+1/2$ units of M2 brane where the 1/2 (in the double cover S^7) charge shift is realized by turning on the discrete torsion as in Eq. (2.10) while the singularity at π having $-1/8$ units of M2 brane charge. Thus we wish to classify the O2 planes in terms of distinct fluxes for the four-form G_4 on RP^7 at the origin which correspond to distinct strong coupling limits for O2 planes [17]. The relevant cohomology corresponds to the possible choices of discrete torsion and is given by

$$H^4(RP^7,Z)=Z_2. \tag{2.11}$$

Consequently, $O2^-$ plane flows to the case without discrete torsion while $\widetilde{O2}^+, O2^+$ flow to the case with discrete torsion.

Since the $SO(N)/Sp(2N)$ gauge theories for large N can be distinguished by the sign of the string RP^2 diagram, this can be classified by the discrete torsion of the B_{NS} field [13]. Note that the orbifolding in R^8/Z_2 does not act on C_3 but it acts on x_{10}. Since the $B_{\mu\nu}$ field of type IIA string theory corresponds in M theory to $C_{\mu\nu 10}$, Z_2 action in ten dimensions flips the sign of B_{NS}. This means that a cohomology class $[H_{NS}]$ takes values in a twisted integer coefficient $\tilde{\mathbf{Z}}$ where the twisting is determined by an orientation bundle. The relevant cohomology groups measuring the topological types of the fields B_{NS} is given by

$$H^3(RP^7,\tilde{Z})\approx Z_2. \tag{2.12}$$

Thus the relevant cohomology groups measuring the topological types for the O2 planes are given by

$$H^3(RP^7,\tilde{Z})\approx Z_2,\quad H^4(RP^7,Z)\approx Z_2. \tag{2.13}$$

If we denote the values of the cohomologies $H^3(RP^7,\tilde{Z})$ and $H^4(RP^7,Z)$ as (α,β) respectively, we get the topological classification of the three models:

$$O2^-:(\alpha,\beta)=(0,0),\quad \widetilde{O2}^+:(\alpha,\beta)=(0,1),$$

[4]We count the number of brane charges on Z_2 orbifold in the double cover of the Z_2 quotient where the charge of a D2 brane is 1.

[5]We will also not consider nontrivial holonomy [RR $U(1)$ Wilson line] around the eleventh circle as in [17] since the gauge theory has a smooth strong coupling limit in decompactified M theory limit and the holonomy indeed vanishes due to $H^2(RP^7,\tilde{Z})=0$.

$$O2^{+}:(\alpha,\beta)=(1,1). \tag{2.14}$$

Note that the topological type of $O2^{+}$ at $x^{10}=\pi$ is $(\alpha,\beta)=(1,0)$.

C. Various wrapped branes

Now we consider the possibilities of brane wrapping on RP^7 in the M theory. The wrappings of M2 brane unwrapped around x^{10} and M5 brane wrapped around x^{10} are classified by the ordinary (untwisted) homology $H_i(RP^7,\mathbf{Z})$ for wrapped on an *i*-cycle in RP^7 since the two-brane charge is even under the orientifolding operation (it comes from the fact that, in M theory, Z_2 action does not act on the three-form C_3) and the five-brane is dual to the two-brane: (i) unwrapped M2 brane, giving a two-brane in AdS_4, (ii) wrapped on a one-cycle, to give a one-brane in AdS_4, classified by $H_1(RP^7,Z)=Z_2$, The unwrapped M5 brane is not possible. (iii) wrapped on a one-cycle, to give a four-brane in AdS_4, classified by $H_1(RP^7,Z)=Z_2$, (iv) wrapped on a three-cycle, to give a two-brane in AdS_4, classified by $H_3(RP^7,Z)=Z_2$. (v) wrapped on a five-cycle, to give a zero-brane in AdS_4, classified by $H_5(RP^7,Z)=Z_2$. The wrappings of M2 brane wrapped around x^{10} and M5 brane unwrapped around x^{10} are classified by the twisted homology $H_i(RP^7,\tilde{Z})$ for wrapped on an *i*-cycle in RP^7. The wrapping modes that would give one-branes are not possible since $H_1(RP^7,\tilde{Z})=0$. The unwrapped M5 brane (unwrapped around x^{10}) is not possible. (vi) wrapped on a two-cycle, to give a three-brane in AdS_4, classified by $H_2(RP^7,\tilde{Z})=Z_2$, (vii) wrapped on a four-cycle, to give a one-brane in AdS_4, classified by $H_4(RP^7,\tilde{Z})=Z_2$.

The k units of KK momentum mode around the eleventh circle S^1 can be identified as k D0-branes which is charged Bogomol'nyi-Prasad-Sommerfield (BPS) particles.[6] In the three dimensional gauge theory context [22], a new scalar modulus appears as the dual of a photon, the magnetic scalar photon, coming from dualizing the vector in three dimensions. This corresponds to the expectation value of the localized eleven dimensional coordinate of a M2 brane. Then the effect of D0 branes exchange between M2 branes can be captured by instanton effects in 3-dimensional SYM and renders it $SO(8)$ invariant [20].

According to the similar arguments done in type IIB description [13] and type IIA description [15], one can derive a topological restriction on the brane wrappings on RP^7 just described. In particular, since the topological restriction coming from the holonomy of the connection A_{RR} on the $U(1)$-bundle would not be considered for the reason explained in the previous footnote, it is sufficient only to consider the discrete torsion θ_{NS} of the field B_{NS}. We will show that the description of brane wrappings on RP^7 is consistent with the topological restriction coming from the RR discrete torsion θ_{RR}, which should vanish in our case. In the case (ii) and (iii), there is no restriction on wrapping of M2 and M5 branes on $RP^1\subset RP^7$, since in this case RP^2 cannot even be deformed into the M2 or M5 brane. In the case (iv), the M5 brane can be wrapped on RP^3, to make a two-brane in AdS_4, only if $\theta_{NS}\neq 0$, since $H^2(RP^3,Z)=Z_2$. In the case (v), the M5 brane can be wrapped on RP^5, to make a zero-brane in AdS_4, only if $\theta_{NS}\neq 0$, since $H^2(RP^5,Z)=Z_2$. In the case (vi), there is no restriction on wrapping of M5 branes on $RP^2\subset RP^7$, to make a three-brane in AdS_4, since $H^2(RP^2,\tilde{Z})=Z$. In the case (vii), the M5 brane can be wrapped on RP^4, only if $\theta_{RR}=0$, since $H^2(RP^4,\tilde{Z})=0$.

III. GAUGE THEORY AND BRANES ON RP^7

A. Domain walls

Let us consider the objects in $AdS_4\times S^7$ and $AdS_4\times RP^7$ that look like two-branes in the four noncompact dimensions of AdS_4. Since the AdS_4 has three spatial dimensions, the two-brane could potentially behave as a domain wall, with the ''jumping'' as one crosses the two-brane. In $AdS_4\times S^7$ and $AdS_4\times RP^7$, the only such objects are M2 brane in the case (i) and the M5 brane in the case (iv) in Sec. II C. Note that, in the case of $AdS_4\times S^7$, the M5 brane cannot wrap on a five-cycle in S^7 since $H_5(S^7,Z)=0$, so does not give rise to a domain wall in $SU(N)$ gauge theory.

Since the two-brane is the electric source of the four-form field G_4, the integrated four-form flux over S^7 or RP^7 jumps by one unit when one crosses the two-brane. This means that the gauge group of the boundary conformal field theory can change, for example, from $SU(N)$ on one side to $SU(N\pm 1)$ on the other side for $AdS_4\times S^7$. For $AdS_4\times RP^7$, if one is crossing the M2 brane, it changes from $SO(N)$ to $SO(N\pm 2)$ or from $Sp(N/2)$ to $Sp(N/2\pm 1)$ since the M2 brane charge changes by two units on double cover.

The similar situation also occurs in the case of the M5 brane wrapped on $RP^3\subset RP^7$ to make a two-brane. Let P and Q be points on opposite sides of the two-brane. Let X be the three-manifold $X=T\times RP^4$, with T a path from P and Q, intersecting the two-brane once. Since a generic RP^3 and RP^4 in RP^7 have one point of intersection, X generically intersects the M5 brane at one point. The boundary of X is the union of the two-manifolds $P\times RP^4$ and $Q\times RP^4$. Although it causes no change of the gauge group in the boundary theory, this may have an important effect on instanton corrections in field theory.

B. The baryon vertex in $SO(N)/Sp(2N)$

The baryon vertex in $SU(N)$ was obtained by wrapping a M5 brane over S^5. By analogy, one expects that the baryon vertex in $SO(N)$ or $Sp(2N)$ will consist of a M5 brane wrapped on RP^5. If we are considering $SO(2k)$ gauge theory, there are k units of five-form flux on RP^5 when the M5 brane wraps once on RP^5. But there is no gauge invariant combination, in $SO(2k)$ gauge theory, of k external quarks to obtain a ''baryon vertex.'' The baryon vertex of $SO(2k)$ gauge theory should couple $2k$ external quarks, not k of them.

[6] Note that the D0 brane charge is odd under the Z_2 action since $A_\mu^{RR}=G_{\mu 10}$ and the Z_2 flips the orientation of S^1 and thus the RR $U(1)$ gauge field should be considered as twisted one-form.

Let Φ be the map of M5 brane worldvolume X to $AdS_4\times RP^7$. We must impose the condition that the B_{NS} fields should be topologically trivial when pulled back to X as implied in deriving the topological restrictions on the brane wrapping in Sec. II. If we choose the M5 brane topology as S^5, the topological triviality of the field B_{NS} is automatically obeyed since $H^3(S^5,Z)=0$, the AdS baryon vertex thus exists regardless of the gauge group of the boundary theory. Then the map $\Phi:S^5\rightarrow RP^5$ gives the degree two map, in other words, S^5 wraps twice around RP^5. Thus we can obtain the correct baryon vertex coupling $2k$ quarks.

In $Sp(k)$ gauge theory, a baryon vertex can decay to k mesons [26]. Thus, one may expect no topological stability for the AdS_4 baryon vertex when $\theta_{NS}\neq 0$. However, since the stability of baryon in $SO(k)/Sp(k)$ gauge theory actually can be encoded by the homology, $H_5(RP^7,Z)=Z_2$, which implies the topological stability of a baryonic charge, we get the field theory result. We also should consider the possibility on an existence of nontrivial torsion class of the B_{NS} fields due to the topology of the M5 brane world volume X, which is denoted as $W\in H^3(X,Z)$ [13]. Then this means that the correct global restriction is not that $i^*([H_{NS}])=0$ but rather that

$$i^*([H_{NS}])=W, \tag{3.1}$$

where i is the inclusion of X in spacetime and $[H_{NS}]$ the characteristic class of the B_{NS} field. A possible W can be determined by using the ''connecting homomorphism'' in an exact sequence of cohomology groups from the second Stieffel-Whitney class $w_2(X)\in H^2(X,Z_2)$, which means that the proper global restriction is $i^*([H_{NS}])=W$, i.e., $\theta_{NS}\neq 0$ which is also consisitent with the topological anaylsis we did last section. If the baryon vertex decays via compact six-manifold X, we will find that $W\neq 0$ since $w_2(X)\neq 0$. Consequently, the brane decay via compact sixfold X is possible only in $Sp(k)$ gauge theory.

In $SO(N)$ gauge theory where N is even or odd, super Yang-Mills theory with 16 supercharges actually has $O(N)$ symmetry, not just $SO(N)$. The generator τ of the quotient $O(N)/SO(N)=Z_2$ behaves as a global symmetry. Since the baryon is odd under τ, it cannot decay to mesons which is even under τ. If there is a Pfaffian-like state which is odd under τ, the possible decay channel may be mesons plus a Pfaffian as in [13]. However, in our case, there is no definite candidate being role of the Pfaffian. It is well known [26] that two baryons in $SO(N)$ gauge theory can annihilate into N mesons. This is also consistent with the fact that, in $O(N)$, a product of two epsilon symbols can be rewritten as a sum of products of N Kronecker deltas. Their annihilation can be realized by the similar (and more simple) process to the pair annihilation of two fat strings discussed in [13]. That is, two identical M5 branes, whose worldvolumes are of the form $C\times \mathbf{S}^5$ and $C'\times \mathbf{S}^5$ where C and C' are timelike path, collide at any time where C and C' coincide. These results imply that the decay of baryon in $SO(N)/Sp(N)$ gauge theory should belong to the element of $H_5(Y,Z)=Z_2$. When we consider $Y=RP^7$, the baryon vertex is stable since $H_5(RP^7,Z)=Z_2$.

IV. DISCUSSION

To summarize, for $SU(N)$ theory, we interpreted the baryon vertex as a wrapped M5 brane in S^7. When we go $SO(N)/Sp(2N)$ theory, R^8/Z_2 orbifold singularity was crucial to understand M theory realization of three types of O2 plane. We constructed the possible brane wrappings on RP^7 and determined their topological restrictions in each case. According to this classification, it was possible to interpret various wrapping branes on RP^7 in terms of domain walls and the baryon vertex where the topological properties on RP^7 are used.

It was pointed out by Witten [27] that the infrared limit of SYM on R^3 related to $\mathcal{N}=8$ SCFT is quite subtle and there may be suprising features in the topology in the infrared, such as conservation laws that hold in the infrared but not exactly. It may become important in the limit to consider the nonperturbative instanton corrections in SYM theory, which is essential to recover $SO(8)$ invariance or eleven dimensional Lorentz invariance [20].

It was observed by Sethi [17] that M-theory realization of O2 planes gives an amusing interpretation on the shift of membrane charge by the discrete torsion. This interesting phenamenon may be more clearly understood in terms of the method applied to O4 planes by Gimon [28] where, applying the T-S-T transformations, the O4 planes are related to O3 planes which is more well understood. By applying the same strategy, one can relate the O2 planes to O3 planes by the T-S-T transformations where T-duality is taken along the one direction of the O3 plane. We hope this work will be accomplished in the near future and provide a new understanding on O2 planes.

Klebanov and Witten [29] found $AdS_5\times T^{1,1}$ model which is an example of holographic theory on a compact manifold which is not locally S^5 and the corresponding quantum field theory cannot be obtained from the projection of maximal $\mathcal{N}=4$ theory. See also recent papers [30]. It is well known that there exist various types of seven dimensional compact Einstein manifold X_7 which is not locally S^7. It would be interesting to study whether one can find wrapping branes over cycles of X_7 and discuss their field theory interpretation.

Note added. After this work was finished, we found Ref. [31] which treats a related subject.

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